SEARCH @KELEME_2013 ON

TELEGRAM AND JOIN US.

ፍላጎ

ON TELEGRAM

@keleme_2013

DEBRE MARKOS UNIVERSITY
COLLEGE OF NATURAL AND COMPUTATIONAL SCIENCES
DEPARTMENT OF MATHEMATICS
APPLIED MATHEMATICS I (MATH 1042) MID EXAM

Maximum Marks (30%) *Date: 01/07/2014 E.C*
Time Allowed: 2:15 hrs

Name: Answer Key **D.No:** Tech **Section:** ____

Instructions:-

- Attempt All Questions Accordingly.
- Check that the Exam Booklet Contains **6 different Pages.**
- Do not use any **rough paper** or **mobile Phone** during the exam Session.

Part I: For each of the following questions write "True" if the statement is correct and "False" if the statement is incorrect on space provided. (1 point each)

False 1) For any two vectors A and B in the same space, $\|A + B\| = \|A\| + \|B\|$

True 2) If two position vectors have the same terminal point, then their norms are equal.

True 3) If A and B are invertible square matrices of the same size, then AB is invertible.

True 4) If A is mxn and B is nxp matrices, then $(AB)^t = B^tA^t$.

False 5) For a square matrix A, if $AA^t = A^tA$, then A is orthogonal matrix.

True 6) Assume A is 3x3 matrix such that $A^2 = A$, then $|A^5| = |A|$.

True 7) The homogeneous linear system $AX = 0$ is consistent for any $m \times n$ matrix A.

False 8) For any matrices A & B, if $BA = 0$, then matrix $A = 0$ or matrix $B = 0$.

Part II: For each of the following Questions write the correct answer only on the space provided. (1 point each)

1) Let $A = (2, -2, 1)$ *and* $B = (-3, 0, 4)$, then $Comp_B^A =$ $-\frac{2}{5}$ and $Proj_A^B =$ $\frac{-2}{9}(2, -2, 1)$

2) If A and B are perpendicular unit vectors, then $\|A - B\| =$ $\sqrt{2}$

3) The area of the triangle having vertices P = (3,-2,1), Q=(7,-3,4) and R=(5,1,0) is $\frac{3\sqrt{10}}{2}$

4) The rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -1 & 2 & -3 \end{bmatrix}$ is 2

5) The distance from the point(3,1,2) to the plane $2x + 3y - 5z = 4$ is $\frac{5}{\sqrt{38}}$ or $\frac{5\sqrt{38}}{38}$

6) Let A & B are 2x2 matrices, such that $|A| = -4, |B| = 3$, *then* $|2AB^tB| =$ -144

7) Give the value(s) of m for which the matrix, $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & m \\ 0 & 2 & -m \end{bmatrix}$ is non-singular. $m \neq 4$

1

DEBRE MARKOS UNIVERSITY
NATURAL AND COMPUTATIONAL SCIENCES
DEPARTMENT OF MATHEMATICS
[illegible] (MATH 1042) MID EXAM

2

Part III: For each of the following Questions write all the necessary steps clearly and neatly on the space provided.(15%)

1. Let L_1 and L_2 be two lines in a plane π.

   If $L_1: (4,-4,-10) + t_1(2-p, p, 3+p)$ *and* $L_2: (11,5,2) + t_2(-p, 2-p, -3+p)$ are orthogonal, then find

   a) the possible value(s) of p. ***(1 point)***

   b) the equation of the plane π, in the form of $ax + by + cz = d$. ***(2 points)***

Solution

Given, $L_1: (4,-4,-10) + t_1(2-p, p, 3+p)$ and
$L_2: (11, 5, 2) + t_2(-p, 2-p, -3+p)$

Now, $V_1 = (2-p, p, 3+p)$ and $V_2 = (-p, 2-p, -3+p)$ } 0.5

a) Since, L_1 & L_2 are Orthogonal, then $V_1 \cdot V_2 = 0$

$\Rightarrow (2-p, p, 3+p)\cdot(-p, 2-p, -3+p) = 0$

$\Rightarrow (2-p)(-p) + p(2-p) + (3+p)(-3+p) = 0$

$\Rightarrow -2p + p^2 + 2p - p^2 + p^2 - 9 = 0$ } 0.5

$\Rightarrow p = \pm 3$ // but p can't be -3.

b) To find the equation of the plane 1st find the point of intersection of L_1 & L_2, Say (x_0, y_0, z_0). We get at $p = 3$.

$V_1 = (-1, 3, 6)$ and $V_2 = (-3, -1, 0)$.

$$\left.\begin{cases} 4 - t_1 = 11 - 3t_2 \\ -4 + 3t_1 = 5 - t_2 \\ -10 + 6t_1 = 2 \end{cases} \Rightarrow \begin{cases} t_1 = 2 \\ t_2 = 3 \end{cases} \text{. Hence} \right\} 0.5$$

$P = (x_0, y_0, z_0) = (2, 2, 2)$

Again, $N = V_1 \times V_2 = \begin{vmatrix} i & j & k \\ -1 & 3 & 6 \\ -3 & -1 & 0 \end{vmatrix} = (6, -18, 10)$ } 0.5

Hence, Equation of the plane $P_0P \cdot \vec{N} = 0$, where $P = (x, y, z)$
$(x-2, y-2, z-2)\cdot(6, -18, 10) = 0$ } 0.5 0.5

$6(x-2) - 18(y-2) + 10(z-2) = 0$ $(6x - 18y + 10z = -4)$

$6x - 12 - 18y + 36 + 10z - 20 = 0$

$\Rightarrow 6x - 18y + 10z = -4$ $\Rightarrow 3x - 9y + 5z = -2$

DEBRE MARKOS UNIVERSITY
NATURAL AND COMPUTATIONAL SCIENCES
DEPARTMENT OF MATHEMATICS
...TICS I (MATH 1042) MID EXAM
Date: 01/07/2014 E.C
...llowed: 2:15 hrs

3

2. Let V={(1,−3,1), (−1,4,−2), (5,−2,3)}
   a. Show that the vectors in V form a basis of $\mathbb{R}^3$. (2 points)
   b. Determine the coordinate vector of $u = (12, 3, 9)$ with respect to the given basis. (1 point)

Sol:

Let $v_1 = (1,-3,1)$, $v_2 = (-1,4,-2)$ and $v_3 = (5,-2,3)$

(a) To show $B = \{v_1, v_2, v_3\}$ form a basis for $\mathbb{R}^3$.

(i) let $c_1, c_2, c_3 \in \mathbb{R}$ such that $c_1v_1 + c_2v_2 + c_3v_3 = 0$ whenever, $c_1 = c_2 = c_3 = 0$.

Now, $c_1(1,-3,1) + c_2(-1,4,-2) + c_3(5,-2,3) = (0,0,0)$

$\Rightarrow (c_1 - c_2 + 5c_3, -3c_1 + 4c_2 - 2c_3, c_1 - 2c_2 + 3c_3) = (0,0,0)$.

$$\Rightarrow \begin{cases} c_1 - c_2 + 5c_3 = 0 \\ -3c_1 + 4c_2 - 2c_3 = 0 \\ c_1 - 2c_2 + 3c_3 = 0 \end{cases} \Rightarrow \begin{bmatrix} 1 & -1 & 5 \\ -3 & 4 & -2 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

Now, $|A| = \begin{vmatrix} 1 & -1 & 5 \\ -3 & 4 & -2 \\ 1 & -2 & 3 \end{vmatrix} = 1\begin{vmatrix} 4 & -2 \\ -2 & 3 \end{vmatrix} + 1\begin{vmatrix} -3 & -2 \\ 1 & 3 \end{vmatrix} + 5\begin{vmatrix} -3 & 4 \\ 1 & -2 \end{vmatrix} = (12-4) + (-9+2) + 5(6-4)$

$\Rightarrow |A| = 8 - 7 + 10 = 11 \neq 0$.

Hence, the system has only trivial sol'n, $(c_1, c_2, c_3) = (0,0,0)$

That is $c_1 = c_2 = c_3 = 0$ is the only solution.

Therefore, v_1, v_2 and v_3 are linearly Independent.

(ii) To show $B = \{v_1, v_2, v_3\}$ generates $\mathbb{R}^3$.

let $(x,y,z) \in \mathbb{R}^3$. Then, $(x,y,z) = c_1v_1 + c_2v_2 + c_3v_3$

$\Rightarrow (x,y,z) = c_1(1,-3,1) + c_2(-1,4,-2) + c_3(5,-2,3)$

$$\Rightarrow \begin{cases} c_1 - c_2 + 5c_3 = x \\ -3c_1 + 4c_2 - 2c_3 = y \\ c_1 - 2c_2 + 3c_3 = z, \end{cases} \text{ take } A = \begin{bmatrix} 1 & -1 & 5 \\ -3 & 4 & -2 \\ 1 & -2 & 3 \end{bmatrix}, X = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \text{ and}$$

$B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$. Now, $\begin{bmatrix} 1 & -1 & 5 \\ -3 & 4 & -2 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

From (1) we get $C_1 = C_2 - 5C_3 + x$ ---(4)

Substitute (4) on to (2) & (3), we get

$$\begin{cases} C_2 + 13C_3 = 3x + y \text{ ---(5)} \\ -C_2 - 2C_3 = -x + z \text{ ---(6)} \end{cases}$$

$\Rightarrow 11C_3 = 2x + y + z \Rightarrow C_3 = \frac{2x+y+z}{11}$ ---- (7)

Substitute (7) on to (5) we get,

$$C_2 = \frac{7x - 2y - 13z}{11} \text{ ----(8)}$$

Again, Substitute (7) & (8) on to (4), we get

$$C_1 = \frac{8x - 7y - 18z}{11} \text{ ---(9)}$$

Then, any $(x, y, z) \in \mathbb{R}^3$ can be written as a linear combination of v_1, v_2 and v_3 as:

$(x, y, z) = c_1v_1 + c_2v_2 + c_3v_3 = \left(\frac{8x-7y-18z}{11}\right)v_1 + \left(\frac{7x-2y-13z}{11}\right)v_2 + \left(\frac{2x+y+z}{11}\right)v_3$

Therefore, $B = \{v_1, v_2, v_3\}$ generates $\mathbb{R}^3$

b) To find the coordinate vector of $u = (12, 3, 9)$, take, $x = 12$, $y = 3$ and $z = 9$. We get

$$(c_1, c_2, c_3) = \left(\frac{8x-7y-18z}{11}, \frac{7x-2y-13z}{11}, \frac{2x+y+z}{11}\right)$$

$$\Rightarrow (c_1, c_2, c_3) = \left(\frac{-87}{11}, \frac{-39}{11}, \frac{36}{11}\right)$$

3. Solve the following system of equations using the method specified.

a. $\begin{cases} x+y+z=1 \\ 3x+3z=2 \\ -2x+3y=3 \end{cases}$ (Use Cramer's Rule) (3 points)

5

Soln

Let $A=\begin{bmatrix}1&1&1\\3&0&3\\-2&3&0\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$ and $B=\begin{bmatrix}1\\2\\3\end{bmatrix}$. Then, the matrix form/Augmented matrix $AX=B$ ①

$$\begin{bmatrix}1&1&1\\3&0&3\\-2&3&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$$

Now, $|A|=1\begin{vmatrix}0&3\\3&0\end{vmatrix}-1\begin{vmatrix}3&3\\-2&0\end{vmatrix}+1\begin{vmatrix}3&0\\-2&3\end{vmatrix}=(-9)-(0-(-6))+(9-0)$

$=-9-6+9=-6\neq 0$

$D=$

Using Cramer's Rule $x_i=\frac{D_i}{D}$, where $D=|A|$ & $D_i=$ det(A:

Now, $x_1=x=\dfrac{\begin{vmatrix}1&1&1\\2&0&3\\3&3&0\end{vmatrix}}{-6}=\frac{1}{-6}\left(1\begin{vmatrix}0&3\\3&0\end{vmatrix}-1\begin{vmatrix}2&3\\3&0\end{vmatrix}+1\begin{vmatrix}2&0\\3&3\end{vmatrix}\right)$

$\Rightarrow x=\frac{-1}{6}(-9-(-9)+(6-0))=\frac{-1}{6}(6)=-1$ } 0.5

$y=\dfrac{D_2}{D}=\dfrac{\begin{vmatrix}1&1&1\\3&2&3\\-2&3&0\end{vmatrix}}{-6}=\frac{-1}{6}\left(1\begin{vmatrix}2&3\\3&0\end{vmatrix}-\begin{vmatrix}3&3\\-2&0\end{vmatrix}+\begin{vmatrix}3&2\\-2&3\end{vmatrix}\right)$ } 0.5

$\Rightarrow y=\frac{-1}{6}(-9-(0-(-6))+(9-(-4)))=\frac{-1}{6}(-15+13)=\frac{1}{3}$

$z=\dfrac{D_3}{D}=\dfrac{\begin{vmatrix}1&1&1\\3&0&2\\-2&3&3\end{vmatrix}}{-6}=\frac{-1}{6}\left(1\begin{vmatrix}0&2\\3&3\end{vmatrix}-\begin{vmatrix}3&2\\-2&3\end{vmatrix}+\begin{vmatrix}3&0\\-2&3\end{vmatrix}\right)=\frac{-1}{6}(-6-13+9)$ } 0.5

$\Rightarrow z=\frac{-1}{6}(-18+9)=\frac{10}{6}=\frac{5}{3}$ } 0.5

$\therefore S.S=(x,y,z)=\left(-1,\frac{1}{3},\frac{5}{3}\right)$ //

b. $\begin{cases} -3x+2y-6z=6 \\ 5x+7y-5z=6 \\ x+4y-2z=8 \end{cases}$ (Use Matrix Inversion Method). *(3 points)* **6**

Soln

let $A=\begin{bmatrix} -3 & 2 & -6 \\ 5 & 7 & -5 \\ 1 & 4 & -2 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B=\begin{bmatrix} 6 \\ 6 \\ 8 \end{bmatrix}$

$A^{-1}=\frac{1}{|A|}(adj(A))$, But $|A|=-3\begin{vmatrix} 7 & -5 \\ 4 & -2 \end{vmatrix}-2\begin{vmatrix} 5 & -5 \\ 1 & -2 \end{vmatrix}+(-6)\begin{vmatrix} 5 & 7 \\ 1 & 4 \end{vmatrix}$ (1)

$\Rightarrow |A|=-3(-14-(-20))-2(-10-(-5))-6(5(4)-7(1))$

$\Rightarrow |A|=-3(-14+20)-2(-10+5)-6(20-7)$

$\Rightarrow |A|=-3(6)-2(-5)-6(13)=-18+10-78$

$=-86\neq 0$.

$adj(A)=C^t$, But $C=\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$,

where $C_{11}=M_{11}=6$, $C_{12}=-M_{12}=-\begin{vmatrix} 5 & -5 \\ 1 & -2 \end{vmatrix}=-5$, $C_{13}=\begin{vmatrix} 5 & 7 \\ 1 & 4 \end{vmatrix}=13$

$C_{21}=-M_{21}=-\begin{vmatrix} 2 & -6 \\ 4 & -2 \end{vmatrix}=-(-4+24)=-20$, $C_{22}=\begin{vmatrix} -3 & -6 \\ 1 & -2 \end{vmatrix}=12$

$C_{31}=\begin{vmatrix} 2 & -6 \\ 7 & -5 \end{vmatrix}=-10+42=32$, $C_{32}=-\begin{vmatrix} -3 & -6 \\ 5 & -5 \end{vmatrix}=$ $C_{23}=-\begin{vmatrix} -3 & 2 \\ 1 & 4 \end{vmatrix}=14$

$=-(15+30)=-45$

$C_{33}=\begin{vmatrix} -3 & 2 \\ 5 & 7 \end{vmatrix}=-31$

$adj(A)=\begin{bmatrix} 6 & -20 & 32 \\ 5 & 12 & -45 \\ 13 & 14 & -31 \end{bmatrix}$ (1)

sIPage

$A^{-1}=\frac{1}{|A|}adj(A)=\frac{-1}{86}\begin{bmatrix} 6 & -20 & 32 \\ 5 & 12 & -45 \\ 13 & 14 & -31 \end{bmatrix}$

$=\begin{bmatrix} \frac{-6}{86} & \frac{20}{86} & \frac{-32}{86} \\ \frac{-5}{86} & \frac{-12}{86} & \frac{45}{86} \\ \frac{-13}{} & \frac{-14}{} & \frac{31}{} \end{bmatrix}$ (1)

7

Now, Using Matrix Inversion Method, we have

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-6}{86} & \frac{20}{86} & \frac{-32}{86} \\ \frac{-5}{86} & \frac{-12}{86} & \frac{45}{86} \\ \frac{-13}{86} & \frac{-14}{86} & \frac{31}{86} \end{bmatrix} \begin{bmatrix} 6 \\ 6 \\ 8 \end{bmatrix} \quad (1)$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-36}{86} + \frac{120}{86} - \frac{256}{86} \\ \frac{-30}{86} - \frac{72}{86} + \frac{360}{86} \\ \frac{-78}{86} - \frac{84}{86} + \frac{248}{86} \end{bmatrix} = \begin{bmatrix} \frac{-172}{86} \\ \frac{258}{86} \\ \frac{86}{86} \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

Hence, the Solution is $(x, y, z) = (-2, 3, 1)$

$\xrightarrow{R_2 \to R_2 - R_1} \begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \xrightarrow{R_3 \to R_3 - R_2} \begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, then

the System becomes, $\begin{cases} x+y-2z=0 \\ y-z=0 \\ 0=0 \end{cases} \Rightarrow \begin{cases} x=y=z \end{cases}$ (0.5)

Now, let $x = t \in \mathbb{R} \setminus \{0\}$, then $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ t \\ t \end{bmatrix} = t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

) For $\lambda_2 = -1$, We have $\begin{bmatrix} 1-(-1) & 1 & -2 \\ -1 & 0-(-1) & 1 \\ -2 & 1 & 1-(-1) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 1 & -2 \\ -1 & 1 & 1 \\ -2 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} 2x+y-2z=0 \quad \text{---} (1) \\ -x+y+z=0 \quad \text{---} (2) \\ -2x+y+2z=0 \quad \text{---} (3) \end{cases}$

From (1) & (3) we get $y=0$, Substitute in (2) we get $x = z$

Now, let $x = t \in \mathbb{R} \setminus \{0\}$, then the eigenvector becomes

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ 0 \\ t \end{bmatrix} = t \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. (0.5)

) For $\lambda_3 = 3$, we have. $\begin{bmatrix} 1-3 & 1 & -2 \\ -1 & 0-3 & 1 \\ -2 & 1 & 1-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} -2x+y-2z=0 \\ -x-3y+z=0 \\ -2x+y-2z=0 \end{cases}$

$\begin{bmatrix} -2 & 1 & -2 & 0 \\ -1 & -3 & 1 & 0 \\ -2 & 1 & -2 & 0 \end{bmatrix} \xrightarrow{R_1 \to R_2} \begin{bmatrix} -1 & -3 & 1 & 0 \\ -2 & 1 & -2 & 0 \\ -2 & 1 & -2 & 0 \end{bmatrix} \xrightarrow[R_1 \to -R_1]{R_3 \to R_3 - R_2} \begin{bmatrix} 1 & 3 & -1 & 0 \\ -2 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\xrightarrow{R_2 \to R_2 + 2R_1} \begin{bmatrix} 1 & 3 & -1 & 0 \\ 0 & 7 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, the system becomes $\begin{cases} x+3y-z=0 \\ 7y-4z=0 \\ 0=0 \end{cases}$

Now, $y = \frac{4}{7}z$ and $x = -\frac{5}{7}z$. Let $z = t \in \mathbb{R} \setminus \{0\}$. We get

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{5}{7}t \\ \frac{4}{7}t \\ t \end{bmatrix} = t \begin{bmatrix} -\frac{5}{7} \\ \frac{4}{7} \\ 1 \end{bmatrix}$ // (0.5)

9

4. Find all eigenvalues and corresponding eigenvectors of the matrix,

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 0 & 1 \\ -2 & 1 & 1 \end{bmatrix}$$

8 *(3 points)*

Soln

Given, $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 0 & 1 \\ -2 & 1 & 1 \end{bmatrix}$ to find the eigenvalues & eigenvectors

Let λ be the eigenvalues and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be the non-zero corresponding eigenvectors. Then, $(A - \lambda I_3)X = 0$

The characteristic eqn is $|A - \lambda I_3| = 0$

$$\Rightarrow \left| \begin{bmatrix} 1-\lambda & 1 & -2 \\ -1 & 0-\lambda & 1 \\ -2 & 1 & 1-\lambda \end{bmatrix} \right| = 0$$

1.5

$$\Rightarrow (1-\lambda)\begin{vmatrix} -\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} - 1\begin{vmatrix} -1 & 1 \\ -2 & 1-\lambda \end{vmatrix} - 2\begin{vmatrix} -1 & -\lambda \\ -2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow -\lambda^3 + 2\lambda^2 + 3\lambda = 0$$

$$\Rightarrow -\lambda(\lambda^2 - 2\lambda - 3) = 0$$

$$\Rightarrow \lambda = 0 \text{ or } \lambda^2 - 2\lambda - 3 = 0$$

$$\Rightarrow \lambda = 0 \text{ or } (\lambda+1) = 0 \text{ or } \lambda - 3 = 0$$

$\Rightarrow \lambda_1 = 0, \lambda_2 = -1$ and $\lambda_3 = 3$.

Now, to find the eigenvectors, solve $(A - \lambda I_3)X = 0$

$$\begin{bmatrix} 1-\lambda & 1 & -2 \\ -1 & -\lambda & 1 \\ -2 & 1 & 1-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(i) For $\lambda_1 = 0$, $\begin{bmatrix} 1 & 1 & -2 \\ -1 & 0 & 1 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} x + y - 2z = 0 \\ -x + z = 0 \\ -2x + y + z = 0 \end{cases}$

Now, $\left[\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ -1 & 0 & 1 & 0 \\ -2 & 1 & 1 & 0 \end{array}\right] \xrightarrow{R_2 \to R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ -2 & 1 & 1 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 + 2R_1} \left[\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 3 & -3 & 0 \end{array}\right]$